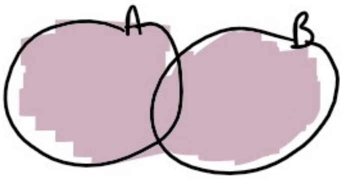
Lezione 20-09

mercoledì 20 settembre 2023 09:06

OPERAZIONI TRA INSIEMI

(1) **UNIONE**

Dati A, B insiemi, $A \cup B := \{x \mid x \in A \text{ oppure } x \in B\}$



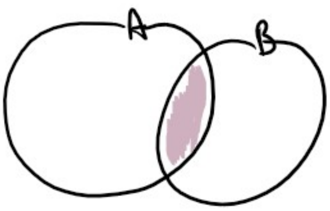
Es - $A = \{a, f, g, h\}$

$B = \{a, b, c, d\}$

$A \cup B = \{a, f, g, h, b, c, d\}$

(2) **INTERSEZIONE**

Dati A, B, $A \cap B := \{x \mid x \in A \text{ e } x \in B\}$



Es - A e B come prima

$A \cap B = \{a\}$

Oss - $\cap$ e $\cup$ sono <u>commutative</u> ($A \cap B = B \cap A$ e $A \cup B = B \cup A$)

$\cap$ e $\cup$ sono <u>associative</u> ($A \cup (B \cup C) = (A \cup B) \cup C$ e

$A \cap (B \cap C) = A \cap (B \cap C)$)

$A_1 \cup \dots \cup A_n = \{x \mid x \in A_1 \text{ oppure } \dots \; x \in A_n\}$

$A_1 \cap \dots \cap A_n = \{x \mid \underbrace{x \in A_i \;\forall i = 1, \dots, n}_{}\}$

X appartiene a tutti $A_1, \dots, A_n$

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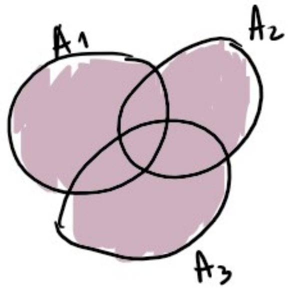
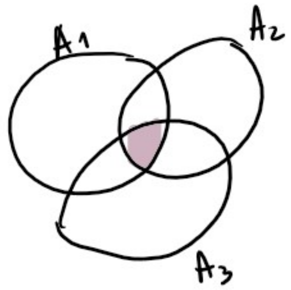




$A_1 = \{1, 2, 3\}$

$A_2 = \{1, 3, 5\}$

$A_3 = \{4, 5, 3\}$

$A_1 \cup A_2 \cup A_3 = \{1, 2, 3, 4, 5\}$

$A_1 \cap A_2 = \{1, 3\}$

$A_1 \cap A_3 = \{3\}$

$A_2 \cap A_3 = \{3, 5\}$

$A_1 \cap A_2 \cap A_3 = \{3\}$

NOTAZIONE: $A_1 \cup \dots \cup A_n = \bigcup_{i=1}^{n} A_i$

$A_1 \cap \dots \cap A_n = \bigcap_{i=1}^{n} A_i$

Es - $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \dots, \{n\}, \dots$

$\bigcup_{n=1}^{\infty} \{n\} = \{1, 2, 3, 4, 5, \dots\} = \mathbb{N}$

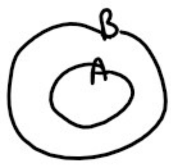
Es - $A_n = [0, n] \subseteq \mathbb{R}$

$\bigcap_{n=1}^{\infty} A_n = [0, 1]$

$[0, 1] \subseteq [0, n] \ \forall n$

OSS - (1) $A \subseteq B$ $\quad A \cap B = A$

$A \cup B = B$



(2) $A \cap B \subseteq A$

$A \cap B \subseteq B$

$A \cap B = \{x \mid x \in A \text{ e } x \in B\} \subseteq A$

$\subseteq B$

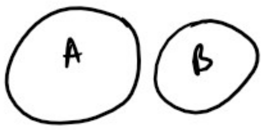
$A \cap B \subseteq B$ $\qquad$ $A \cap B = \{ x \mid x \in A \text{ e } x \in B \} \subseteq A$, $\subseteq B$

$A \subseteq A \cup B$ $\qquad$ $A \subseteq A \cup B = \{x \mid x \in A \text{ o } x \in B\}$

$B \subseteq A \cup B$ $\qquad$ $A \cup B \supseteq A$

(3) Se A e B hanno intersezione vuota, li chiamiamo DISGIUNTI

$A \cap B = \phi$



$A = \{1, 2, 3\}$

$B = \{a, b, c\}$

$A \cap B = \ ? \ \phi$

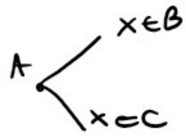
LEMMA

Dati A, B, C, $\qquad \underbrace{A \cap (B \cup C)} = \underline{(A \cap B) \cup (A \cap C)}$

Dim

(i) $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

(ii) $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$



$x \in A \cap (B \cup C) \iff x \in A$ e ($x \in B$ oppure $x \in C$)

$\iff$ ($x \in A$ e $x \in C$) oppure ($x \in A$ e $x \in B$)

$\iff$ ($x \in A \cap C$) oppure ($x \in A \cap B$)

$\iff x \in (A \cap C) \cup (A \cap B)$

$\Rightarrow \quad A \cap (B \cup C) \subseteq (A \cap C) \cup (A \cap B)$

$x \in (A \cap B) \cup (A \cap C) \quad \ldots \quad \ldots \quad x \in A \cap (B \cup C)$

$\square$

PER CASA $\quad$ Dimostrare che

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

(3) DIFFERENZA TRA INSIEMI

A, B insiemi

$A \setminus B = \{ x \in A \mid x \notin B \}$

A, B insiemi

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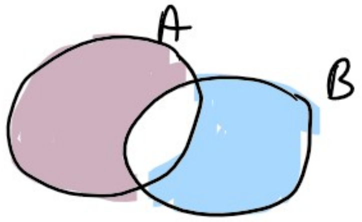
$B \setminus A = \{ x \in B \mid x \notin A \}$



$A = \{1, 2, 3\}$ $\qquad$ $A \setminus B = \{1, 2\}$

$B = \{3, 7, 11\}$ $\qquad$ $B \setminus A = \{7, 11\}$

<u>DEF</u> - $m\mathbb{Z} = \{ ma \in \mathbb{Z} \mid a \in \mathbb{Z} \} = \{ \text{multipli di } m \}$

$m\mathbb{N} = \{ \text{multipli positivi di } m \}$

$2\mathbb{N} = \{2, 4, 6, 8, \dots\}$ $\qquad$ $2\mathbb{N}_0 = \{0, 2, 4, \dots\}$

$2\mathbb{Z} = \{\dots -2, 0, +2, 4, \dots\}$

$3\mathbb{Z} = \{\dots, -9, -6, -3, 0, 3, 6, \dots\}$

$\underset{m}{3} \cdot \underbrace{(-3)}_{a}$

<u>ES</u> - $3\mathbb{Z}, 6\mathbb{Z}$

$3\mathbb{Z} \cup 6\mathbb{Z} = 3\mathbb{Z}$

$6\mathbb{Z} \subseteq 3\mathbb{Z}$ perché $x \in 6\mathbb{Z} \iff \underbrace{x = 6a}$, $a \in \mathbb{Z}$

$x = 3 \cdot 2 \cdot a = 3\,\underline{(2a)}$

$\Rightarrow$ x è anche multiplo di 3 e $x \in 3\mathbb{Z}$

$3\mathbb{Z} \cap 6\mathbb{Z} = 6\mathbb{Z}$

$\underline{\underline{3\mathbb{Z} \setminus 6\mathbb{Z}}} = \{ \text{i multipli di 3 che non sono multipli di 6} \}$

$= \{ -9, +9, -15, +15, 21, -21, 27, -27, \dots, -3, +3, \dots \}$

$= \{ \ldots, -3, +3, \ldots \}$

(4) **PRODOTTO CARTESIANO DI INSIEMI**

A, B insiemi $\qquad A \times B = \{ \underbrace{(a,b)}_{\text{coppie ordinate}} \mid a \in A, \ b \in B \}$

$A = \{ a, b, c \}$

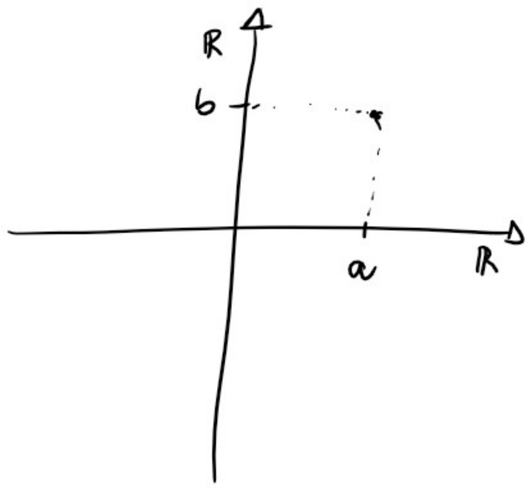
$B = \{ 1, 2 \}$

$A \times B = \{ (a,1), (a,2), (b,1), (b,2), (c,1), (c,2) \}$

$|A \times B| = |A| \cdot |B|$

<u>ES</u> - il piano cartesiano è $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$

$(1,2) \neq (2,1)$



<u>OSS</u> - $A_1 \times A_2 \times A_3 \times \ldots \times A_n$

$= \{ (a_1, a_2, \ldots, a_n) \mid a_1 \in A_1, \ a_2 \in A_2, \ldots, a_n \in A_n \}$

<u>ES</u> $\quad A = \{ 1, 2, 3 \}$

$B = \{ a, b \}$

$C = \{ \pi \}$

$A \times B \times C = \{ (1,a,\pi), (1,b,\pi), (2,a,\pi), (2,b,\pi), (3,a,\pi), (3,b,\pi) \}$

$|A \times B \times C| = |A| \cdot |B| \cdot |C|$

<u>DEF</u> - Una **RELAZIONE** è un sottoinsieme di un prodotto cartesiano.

DEF- Una RELAZIONE è un sottoinsieme di un prodotto cartesiano.

Una relazione tra S e T è $R \subseteq S \times T$.

ES- $A = \{a, b, c\}$

$B = \{1, 2\}$

$R = A \times B$

$R = \phi$

$R = \{(a, 1), (c, 2)\}$

$R = \{(a, 2), (b, 1), (c, 1)\}$

ES- $A = \{$abitanti di salerno$\}$

$R = \{(x, y) \in A \times A \mid x$ è sposato con $y\}$

ES- $S = \{$studenti$\}$

$D = \{$docenti$\}$

$R = \{(s, d) \in S \times D \mid s$ è studente di $d\}$

ES- $\leq \ \subseteq \mathbb{R}^2$

$(x, y) \in \overset{R}{\leq} \iff x \leq y$